LECTURE # 2

Concept of a Function

Historically, the term, function denotes the dependence of one quantity on other quantity. The quantity x is called the independent variable and the quantity y is called the dependent variable.

We write it as $y \quad f \quad x$ and we read y is a function of x.

For example, the equation $y \quad 2x$ defines y as a function of x because <u>each</u> value assigned to x determines <u>unique</u> value of y.

Examples of function

– The area of a circle depends on its radius r by the equation A= r^2 so, we say that A is a function of r.

– The volume of a cube depends on the length of its side x by the equation V= x^3 so, we say that V is a function of x.

– The velocity V of a ball falling freely in the earth's gravitational field increases with time t until it hits the ground, so we say that V is function of t.

– In a bacteria culture, the number n of present after one day of growth depends on the number N of bacteria present initially, so we say that N is function of n.

Function of Several Variables

Many functions depend on more than one independent variable. **Examples**

1) The area of a rectangle depends on its length l and width w by the equation $A = l$ w , so we say that A is a function of l and w.

2) The volume of a rectangular box depends on the length l, width w and height h by the equation $V = l\,w\,h$ so, we say that V is a function of l , w and h.

3) The area of a triangle depends on its base length l and height h by the equation A $\frac{1}{2}\,l\,h$, so we say that A is a function of l and h .

The volume V of a right circular cylinder depends on its radius r and height h by the equation V $r\,h^2$ so, we say that V is a function of r and h.

VALUES OF FUNCTIONS

Example 1 :Consider the function $f \quad x \quad 2x^2-1$, then $f \quad 1 \quad 2\,1 \quad {}^2-1 \quad 1$

$f \quad 4 \quad 2\,4 \quad {}^2-1 \quad 31, f \quad 2 \quad 2 \quad 2 \quad {}^2-1 \quad 7$

$f\,t \quad 4 \quad 2 \quad t \quad 4 \quad {}^2-1 \quad 2t^2 \quad 16t \quad 31$

These are the values of the function at some points.

Example 2 : Now we will consider a function of two variables, so consider the function f $x\,y, \quad x\,y^2 \quad 1$ then $f \quad 2,1 \quad 2\,{}^2\,1\,1 \quad 5, f \quad 1,2 \quad 1$ ${}^2\,2 \quad 1\,3,$

f 0,0 $0^2 0$ 1 1, f 1, 3 1^2 3 1
2,

f $3a\,a$, $3a$ ^2a $19a^3$ 1, $f\,ab\,a$, b ab ^2a b
1 $a\,b^3$ ^{2} $a\,b^2$ ^{3} 1

These are values of the function at some points.

Example 3: Now consider the function $f(x\,y,)$ $x\sqrt{}$ ^{3} xy , then

(a) $f(2,4)$ $2 \sqrt[3]{(2)(4)}$ $2\sqrt[3]{8}$ 2 2 4

(b) $f(t\,t,^2)$ $t\sqrt[3]{()(t\,t^2)}$ $t\sqrt[3]{t^3}$ t t $2t$

(c) $f(x\,x,^2)$ $x\sqrt[3]{()(x\,x^2)}$ $x\sqrt[3]{x^3}$ x x $2x$

(d) $f(2y^2,4\,)y$ $2y^2$ $\sqrt[3]{(2y^2)(4\,)y}$ $2y^2$ $\sqrt[3]{8y^3}$ $2y^2$ $2y$

Example 4: Now again we take another function of three variables

$f(x\,y\,z,,)$ $\sqrt{}$ 1 x_2 — — $\sqrt{}$ $y_2 z_2$ then $f(0,\!\sqrt{1}\,1,$) 1 0

1_2 1_2 1

2 2 2 2 2

Example 5: Consider the function f $x\,y\,z,,$ $xy\,z^{2\,3}$ 3, then at certain points we

have f 2,1,2 $2\;1^2 2^3$ 3 19, f 0,0,0

$0\;0^2 0^3$ 3 3, $f\,a\,a\,a$,, $a\,a^2a^3$

3 a^6 3, $f\,t\,t$,^{2}, t t $t^{2\,2}$ t ^{3} 3 t^8 3,

f 3,1,1 3 1 $^2 1^3$ 3 3 3 0

Example 6 :Consider the function f $x\,y\,z,,$ $x\,y\,z^2$ ^{24}, where $x\,t$ t^3,
$y\,t$ and t^2 $z\,t$ t

a^f $x\,t$,$y\,t$,$z\,t$ $x\,t$ ^{2} $y\,t$

$^2z\,t$ ^{4} t^3 $^2t^{22}$ t^4 t^{14}

$b\,f$ x 0 ,y 0 ,z 0 x 0 ^{2} y

0 ^2z 0 ^{4} 0^3 ^{2} 0^2 ^{2} 0^4 0

Example 7 : Let us consider the function f $x\,y\,z,,$ xyz , then$x$

$f \quad xy, \_x^y \ ,xz \qquad xy \qquad x^y$

$xz \quad xy \quad xy\, z^2 \quad xy$

Example 8 :Let us consider $g\, x\, y\, z \quad , , \quad z \; Sin \; xy \qquad , \; u\, x\, y\, z \quad , , \quad x\, z^{2\,3}$,

xy

$v\, x\, y\, z \quad , , \quad Pxy\; zyz, \; w\, x(, ,) \quad \_\_$, then z

$g\, u\, x\, y\, z \quad , , \quad , v\, x\, y\, z \quad , , \quad , w\, x\, y\, z \quad , , \qquad w\, x\, y\, z \quad , , \quad Sin \; u\, x\, y$

$z \quad , , \quad v\, x\, y\, z, ,$ Now by putting the values of these functions from the above equations, we get $g\, u\, x\, y\, z \quad , , \quad , v\, x\, y\, z \quad , , \quad , w\, x\, y\, z \quad , ,$

$\_\_xy \; Sin \qquad x\, z^{2\,3} \qquad Pxyz \qquad \_\_\frac{x\,y}{z} \; Sin \; P \quad x^3\, yz^4 \quad z$

Example 9 :Consider the function $g\, x\, y \quad , \quad y \; Sin \; x\, y \quad {}^2$ and $u\, x\, y \quad , \quad x\, y^2$

${}^3, v\, x\, y \quad , \quad xy,$

then $g\, u\, x\, y \quad , \quad , v\, x\, y \quad , \quad v\, x\, y \quad , \quad Sin \quad u\, x\, y \quad ,$

${}^2 v\, x\, y \quad ,$

By putting the values of these functions we get

$g\, u\, x\, y \quad , \quad , v\, x\, y \quad , \qquad xy \; Sin \qquad x\, y^2 \quad {}^3 \quad {}^2$

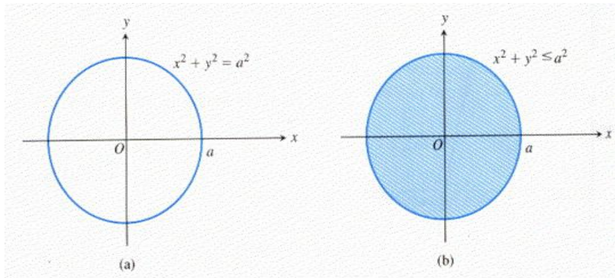
$xy \qquad xy \; Sin \qquad x\, y^5 \quad {}^7$

<u>Function of One Variable</u>: A function f of one real variable x is a rule that assigns a unique real number f(x) to each point x in some set D of the real line.

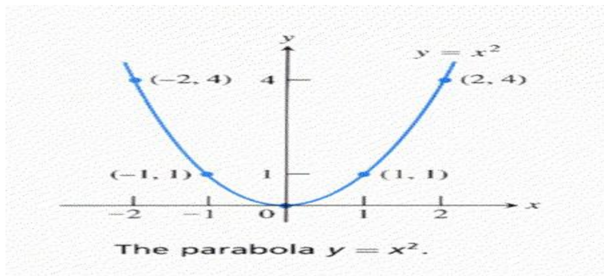
<u>Function of two Variables</u>: A function f in two real variables x and y, is a rule that assigns unique real number $f \quad x\, y,$ to each point (x,y) in some set D of the xy-plane. **<u>Function of three variables</u>:** A function f in three real variables x, y and z, is a rule that assigns a unique real number $f \quad x\, y\, z, ,$ to each point (x,y,z) in some set D of three dimensional space.

<u>Function of n variables</u>: A function f in n variable real variables $x_1, x_2, x_3, \quad , x_n$, is a rule that assigns a unique real number $w \quad f \quad x_1, x_2, x_3, \quad , x_n$ to each point (x_1, x_2, x_3,……, x_n) I n some set D of n dimensional space.

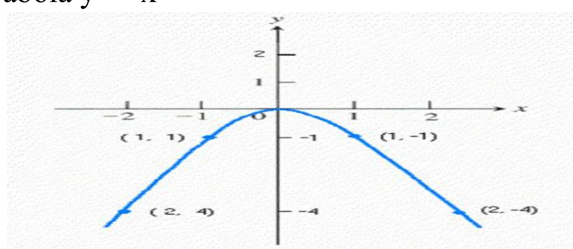
Circles and Disks:

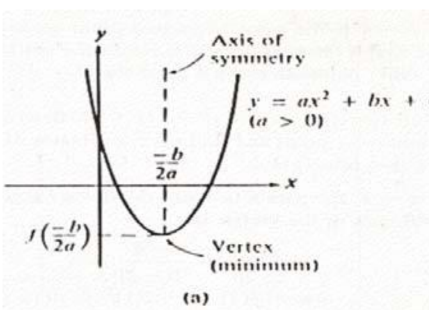


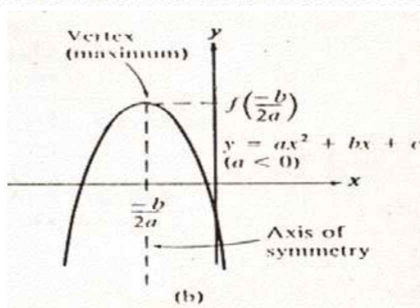
PARABOLA



Parabola $y = -x^2$







General equation of the Parabola opening upward or downward is of the form

$y = f(x) = ax^2 + bx + c.$

Opening upward if $a > 0$, Opening downward if $a < 0$

The x-coordinate of the vertex is given by $x_0 = -\frac{b}{2a}$. So the y-coordinate of the vertex is $y_0 = f(x_0)$. The axis of symmetry is $x = x_0$.

<u>Sketching of the graph of parabola y = ax +bx + c</u>

Finding vertex: x-coordinate of the vertex is given by $x_0 = -\frac{b}{2a}$

The y-coordinate of the vertex is $y_0 = a x_0^2 + b x_0 + c$. Hence vertex is $V(x_0, y_0)$.

Example 10 : Sketch the parabola y x^2 $4x$

Solution:Since a 1 0 because parabola is opening downward. Vertex occurs at

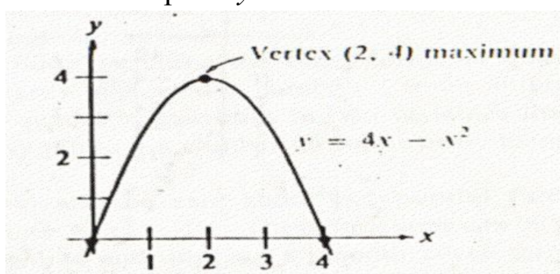
b $4x$ —— 2 Axis of symmetry is the vertical line x 2.

$2a$ 2 1

The y-coordinate of the vertex is y 2^2 4 2 4. Hence vertex is V(2 , 4). The zeros of the parabola (i.e. the point where the parabola meets x-axis) are the solutions to x^2 $4x$ 0 , so x 0 and x 4. Therefore, 0,0 and 4, 0 lie on the parabola.

Also 1,3 and 3, 3 lie on the parabola.

Graph of y x^2 $4x$



Example 11: Sketch the parabola y x^2 $4x$ 3

Solution: Since$a$ 1 0 , parabola is opening upward. Vertex occurs at b 4

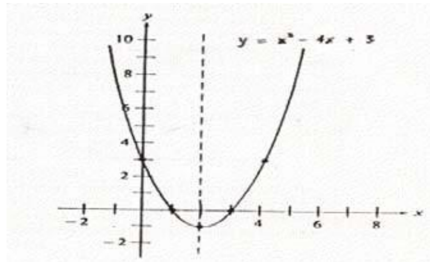
x —— —— 2. Axis of symmetry is the vertical line x 2. The y co-

$2a$ 2 1

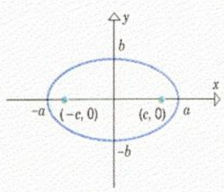
ordinate of the vertex is y 2 ^{2} 4 2 3 1 . Hence vertex is V 2 ,

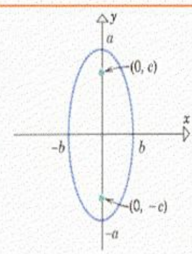
1 . The zeros of the parabola (i.e. the point where the parabola meets x-axis) are the solutions to

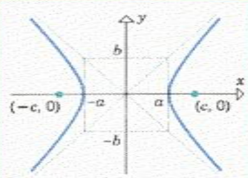
x^2 $4x$ 3 0, so x 1 and x 3 .Therefore 1,0 and 3, 0 lie on the parabola. Also 0 ,3 and 4, 3 lie on the parabola. *Graph of y* x^2 - $4x$ 3



Ellipse

ORIENTATION	DESCRIPTION	STANDARD EQUATION
	• Foci and major axis on the x-axis. • Minor axis on the y-axis. • Center at the origin. • x-intercepts: $\pm a$. • y-intercepts: $\pm b$. • $a \geq b$	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

	• Foci and major axis on the y-axis. • Minor axis on the x-axis. • Center at the origin. • x-intercepts: $\pm b$. • y-intercepts: $\pm a$. • $a \geq b$	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

ORIENTATION	DESCRIPTION	STANDARD EQUATION	ASYMPTOTE EQUATIONS
	• Foci on the x-axis. • Conjugate axis on the y-axis. • Center at the origin.	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$y = \frac{b}{a}x$ $y = -\frac{b}{a}x$

Hyperbola